Dr. C. Arson
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Finite Element Method for Coupled Processes in Elastic Porous Media – Fall 2022

Meeting Times: Monday & Wednesday 9.30am-10.45am
Classroom: Mason 3132
Office hours: Tuesdays and Wednesdays 4pm-5pm via Zoom:
https://gatech.zoom.us/j/95100845654
To meet students' requirements, needs, and comfort levels, office hours will be offered in-person upon request. To schedule in-person office hours, please send an email to Dr. Arson <chloe.arson@ce.gatech.edu>.

Prerequisites: COE undergraduate coursework.
Basic knowledge of FEM recommended, but not mandatory.

Scope of the course: The course explains the formulation of Finite Element Methods (FEM) for elastic porous media, with a special emphasis on coupled thermo-hydro-mechanical processes that occur in soils, rocks and cementitious materials. Applications will focus on climate change and energy storage. First, the course will present the basic principles of the FEM to solve one-dimensional problems of fluid flow and heat transfer in rigid host media, including the variational formulation, space discretization and time discretization. Next, thermo-mechanical and hydro-mechanical equations for two-phase porous media will be introduced after a discussion of the concept of Representative Elementary Volume (REV). 2D space discretization and numerical integration will be explained and applied through simulation and analysis of coupled problems of heat exchangers, consolidation and seepage. The last portion of the course will be dedicated to the effect of capillary pressure (or suction) on the hydro-mechanical properties of porous media. The strong and weak formulations of problems with porous solids filled with two fluids will be established and discussed under a variety of assumptions, e.g., miscible/immiscible fluids, compressible/incompressible solid. Issues of accuracy, convergence and stability will be discussed through case studies, e.g., evapo-transpiration, subsidence, slope stability, desiccation cracks.

Learning Objectives:
1. Describe the set of equations and constraints necessary to solve a poro-mechanical problem.
2. Approximate the solution of Partial Differential Equations (PDEs) by using a variational method.
3. Write the equations of a Finite Element Model (weak form, interpolation functions, element governing equation, global stiffness matrix) for singe-variable problems in 1D - including eigenvalue and time-dependent problems.
4. Solve axis-symmetric, plane strain and plane stress poro-mechanical problems with the Finite Element Method, by using analytical and numerical computations.
5. Interpret numerical errors and convergence problems.
6. Recommend FEM strategies (e.g., meshing, interpolation order, time discretization) for the modeling of coupled processes in porous media.
7. Design a FEM model to understand hydro-mechanical processes in a porous medium subjected to variable boundary conditions imposed by cyclic energy storage or climate change.

Recommended References:

- Dormieux, L., Kondo, D., & Ulm, F. J. *Microporomechanics* (2006), John Wiley & Sons.
- Lewis, R. W., & Schrefler, B. A. *The finite element method in the static and dynamic deformation and consolidation of porous media*, 2nd edition (1998), John Wiley
- Zienkiewicz, O. C., Taylor, R.L. & Zhu, J.Z. *The Finite Element Method: Its Basis and Fundamentals*, 6th edition (2005), Elsevier
- Reddy, J.N. *An Introduction to the Finite Element Method,* 3rd edition (2006), McGraw-Hill

For further reading:

- Coussy, O. *Mechanics and physics of porous solids*, (2011) John Wiley & Sons.
- Zienkiewicz, O. C., Chan, A. H. C., Pastor, M., Schrefler, B. A., & Shiomi, T. *Computational geomechanics* (1999), John Wiley.
- Reddy, J.N. *An Introduction to the Nonlinear Finite Element Analysis* (2004), Oxford University Press

Outline & Schedule: Lecture topics and deadlines will be adjusted as needed in the course of the semester.

<table>
<tr><th>Week</th><th>Deadlines</th><th>Topics</th></tr>
<tr><td>08/22</td><td></td><td rowspan="4">1D FEM for one field variable. Concept of mathematical approximation. Introduction to the FEM: integral formulation, approximation functions, stiffness matrix assembly, boundary conditions, resolution methods and post-processing techniques. Transient problems: time discretization for parabolic and hyperbolic equations. Applications: 1D heat transfer and 1D fluid flow (no deformation).</td></tr>
<tr><td>08/29</td><td></td></tr>
<tr><td>09/05</td><td>HW 1</td></tr>
<tr><td>09/12</td><td></td></tr>
<tr><td>09/19</td><td>HW 2</td><td rowspan="6">1D and 2D FEM for two coupled field variables. Thermo-elasticity and poro-elasticity with one fluid (one phase and one species). Concept of REV, averaging and upscaling techniques. 2D space discretization: triangular and rectangular linear elements, higher order elements, serendipity elements, master elements, coordinate transformation. Numerical integration in 1D and 2D: Newton Cotes quadrature, Gauss quadrature. Stiffness matrix for two coupled field variables: partitioning, staggered resolution. Applications: heat exchangers, consolidation, seepage.</td></tr>
<tr><td>09/26</td><td></td></tr>
<tr><td>10/03</td><td>HW 3</td></tr>
<tr><td>10/10</td><td></td></tr>
<tr><td>10/17</td><td>HW 4</td></tr>
<tr><td>10/24</td><td></td></tr>
<tr><td>10/31</td><td>Project 1</td><td rowspan="6">FEM for unsaturated porous solids. Theory of elasticity for the porous solid filled with two fluids: two phases of the same fluid species vs. two fluid species in the same phase. Variational formulation for the elastic porous solid filled with liquid water and gaseous air under constant temperature. Accuracy, convergence and stability. Applications: subsidence, slope stability. Thermo-poro-elasticity for the unsaturated porous solid: review of equations and discussions through case studies, e.g., evapo-transpiration, desiccation cracks.</td></tr>
<tr><td>11/07</td><td></td></tr>
<tr><td>11/14</td><td>HW 5</td></tr>
<tr><td>11/21</td><td></td></tr>
<tr><td>11/28</td><td></td></tr>
<tr><td>12/05</td><td>Project 2</td></tr>
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Grading: *Final grad*e: F<60%≤D<70%≤C<80%≤B<90%≤A≤100%
Score: 5 HW @ 10% each = 50%. 2 projects @ 25% each = 50%

Exams: No exam in this course.

Homework will include calculus, analysis and programming (mostly in MATLAB). Problems will be similar to, but usually broader in scope than the problems seen in class. Homework will have to be submitted online by the due date, through *Canvas* "Assignments" function. Students needing extra-time must ask permission from the instructor to submit late, before the deadline. A penalty of 5% per late day will apply.

Projects will be individual assignments aimed to use the FEM to solve coupled poro-elastic problems and to analyze Finite Element work published in the academic literature on climate change or energy storage. Students will use the computational software of their choice (e.g. MATLAB, ABAQUS, ANSYS, PLAXIS). For each project, students will write a scientific report about their methods, results and interpretations, prepare a tutorial to explain the procedure to build the FEM model in the software of their choice, and upload sample input and output files to the course website. Dedicated project syllabi will be posted on the course website in due time.

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Academic Integrity: Working in group on homework and projects is allowed (and encouraged). However, each student must write up and turn in their own solutions. Any student suspected of cheating or plagiarizing on a an assignment will be reported to the Office of Student Integrity, who will investigate the incident and identify the appropriate penalty for violations. Georgia Tech aims to cultivate a community based on trust, academic integrity, and honor. Students are expected to act according to the highest ethical standards. For information on Georgia Tech's Academic Honor Code, please visit:
http://www.catalog.gatech.edu/policies/honorcode/ or http://www.catalog.gatech.edu/rules/18/.

Recordings of Class Sessions and Required Permissions: Classes may not be recorded by students without the express consent of the instructor unless it is pursuant to an accommodation granted by the Office of Disability services. Class recordings, lectures, presentations, and other materials posted on Canvas are for the sole purpose of educating the students currently enrolled in the course. Students may not record or share the materials or recordings unless the instructor gives permission.

Dean of Students Office, CARE Center, Counseling Center, Stamps Health Services, and the Student Center: The **CARE Center** and the **Counseling Center**, Stamps Health Services, and the Dean of Students Office will offer both in-person and virtual appointments. Student Center services and operations are available on the **Student Center** website. For more information on these and other student services, contact the Dean of Students or the **Division of Student Life**.

Information Related to Covid-19: Students are expected to be familiar with and abide by the Institute guidelines, information, and updates related to Covid-19. Find campus operational updates, Frequently Asked Questions, and details on campus surveillance testing and vaccine appointments on the **Tech Moving Forward site.**

Accommodations for Students with Disabilities: If you are a student with learning needs that require special accommodation, please contact the Office of Disability Services at (404)894-2563
or http://disabilityservices.gatech.edu/, as soon as possible, to make an appointment to discuss your special needs and to obtain an accommodations letter. Please also e-mail me as soon as possible in order to set up a time to discuss your learning needs.

Diversity Statement: I consider this classroom to be a place where you will be treated with respect, and I welcome individuals of all ages, backgrounds, beliefs, ethnicities, genders, gender identities, gender expressions, national origins, religious affiliations, sexual orientations, ability – and other visible and nonvisible differences. All members of this class are expected to contribute to a respectful, welcoming and inclusive environment for every other member of the class.

Safe Zone Statement: I am a member of a Safe Zone Ally community network, and I am available to listen and support you in a safe and confidential manner. As a Safe Zone Ally, I can help you connect with resources on campus to address problems you may face that interfere with your academic and social success on campus as it relates to issues surrounding sexual orientation and gender identity. I will gladly honor your request to address you by an alternate name or gender pronoun. Please advise me of this preference early in the semester so that I may make appropriate changes to my records. My goal is to help you be successful and to maintain a safe and equitable campus.